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NEW CRITERIA FOR OSCILLATION OF SECOND ORDER NONLINEAR DYNAMIC EQUATIONS WITH DAMPING ON TIME SCALES

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ABSTRACT

The oscillation of solutions of the second order nonlinear damped dynamic equation $(r(t)\psi\,(x(t))x^{\Delta}(t))^{\Delta} + p(t)x^{\Delta}(t) + f(t,x(\tau\,(t))) = 0$ on an arbitrary time scale T is investigated. A generalized Riccati transformation is applied for the study of the Kamenev-type oscillation criteria for this nonlinear dynamic equation. Several new sufficient conditions for the oscillation of solutions are obtained to extend some known results in the literature.

KEYWORDS: Damped Delay Dynamic Equations, Oscillation Criteria, Time Scales

INTRODUCTION

Much recent attention has been given to dynamic equations on time scales, we refer the reader to the landmark paper of S. Hilger [1]. Since then, several authors have expounded on various aspects of this new theory; see the survey paper by Agarwal, Bohner, O'Regan and Peterson [2]. A time scale T is an arbitrary nonempty closed subset of the real numbers R. Thus, $R; Z; N; N_0$, i.e., the real numbers, the integers, the natural numbers, and the nonnegative integers are examples of time scales. On any time scale T, we define the forward and backward jump operators by

$\sigma(t) = \inf\{s \in T,\ s > t\},\ \rho(t) = \sup\{s \in T,\ s < t\}.$

A point t $t \in T,\ t > inf\,T$ is said to be left dense if $\rho(t) = t$, right dense if $t < sup\,T$ and $\sigma(t) = t$, left scattered if $\rho(t) < t$, and right scattered if $\sigma(t) > t.$ A function $f : T \to R$ is called rd-continuous provided that it is continuous at right dense points of T, and its left-sided limits exist (finite) at left-dense points of T. The set of rd-continuous functions is denoted by $C_{rd}(T,R)$. By $C_{rd}^{1}(T,R)$, we mean the set of functions whose delta derivative belongs to $C_{rd}(T,R)$. In recent years, there has been much research activity concerning the oscillation and nonoscillation of solutions of various equations on time scales (see [5],[6],[7]). However, there are few results dealing the oscillation of solutions of delay dynamic equations on time scales [8-13]. Following this trend, we are concerned in this paper with oscillation

for the second-order nonlinear delay dynamic equations of the type

$$(r(t)\psi\,(x(t))x^{\Delta}(t))^{\Delta} + p(t)x^{\Delta}(t) + f(t,x(\tau\,(t))) = 0 \tag{1.1}$$

We assume that

$(\boldsymbol{H_1})$ r,ψ and p are real-valued positive rd-continuous functions defined on T,

$0 \le p(t) \le 1$ and there are two positive constants c_1, c_2 such that $c_1 \le \psi\,(x(t)) \le c_2$.

$(\boldsymbol{H_2})$ $\tau : T \to R$ is strictly increasing, and $\tau(t) \leq t$ and $\tau \to \infty$ as $t \to \infty$

$(\boldsymbol{H_3})$ $f(t, u) \in C_{rd}(T \times R, R)$ satisfies $uf(t, u) > 0$, for $u \neq 0$ and there exists a positive rd-continuous function q defined on T such that $\left|\frac{f(t,u)}{u}\right| \geq q(t)$ for $u \neq 0$.

2. MAIN RESULTS

We need the following lemma for the proof of main results.

Lemma 2.1. *Assume that $x(t)$ is a positive solution of Eq. (1.1) on $[t_0, \infty)_T$. If*

$$\int_0^\infty \frac{e_{\frac{-p(t)}{c_1 r(t)}}(t, t_0)}{r(t)} \Delta t = \infty \tag{2.1}$$

and

$$\int_0^\infty \tau(s) q(s) \Delta s = \infty , \tag{2.2}$$

then there exists $t_1 \in [t_0, \infty)_T$, such that

$(\boldsymbol{i})$ $x^\Delta(t) > 0, (r(t)\,\psi(x(t))x^\Delta(t))^\Delta < 0$ *for* $t_1 \in [t_0, \infty)_T$

$(\boldsymbol{ii})$ $\frac{x(t)}{t}$ *is decreasing.*

Proof. Assume that $x(t)$ is a positive solution of Eq. (1.1) on $[t_0, \infty)_T$. Pick $t_2 \in [t_0, \infty)_T$, such that $x(t) > 0$ and $x(\tau(t)) > 0$ on $[t_2, \infty)_T$. Then without loss of generality we can take $x^\Delta(t) < 0$ for all $t \geq t_2 \geq t_1$. Now from (1.1) we have

$$(r(t)\psi\left(x(t)\right)x^\Delta(t))^\Delta + p(t)x^\Delta(t) = -f\left(t, x^\tau(t)\right) < 0. \tag{2.3}$$

Putting $y(t) = -r(t)\psi\left(x(t)\right)x^\Delta(t)$, then we can write (2.3) in the form

$$-y^\Delta(t) - \frac{p(t)}{r(t)\psi\left(x(t)\right)} y(t) < 0$$

Thus

$$y^\Delta(t) > -\frac{p(t)}{c_1 r(t)} y(t)$$

Therefore

$$y(t) > y(t_2) e_{\frac{-p(t)}{c_1 r(t)}}(t, t_2)$$

i.e.

$$-r(t)\psi\left(x(t)\right)x^\Delta(t) > -r(t_2)\psi\left(x(t_2)\right)x^\Delta(t_2) e_{\frac{-p(t)}{c_1 r(t)}}(t, t_2)$$

$$x^{\Delta}(t) < \frac{r(t_2)\psi\left(x(t_2)\right)x^{\Delta}(t_2)}{\psi\left(x(t)\right)} \frac{e_{\frac{-p(t)}{c_1 r(t)}}(t,t_2)}{r(t)}$$

Then

$$x(t) < x(t_3) + \frac{r(t_2)\psi\left(x(t_2)\right)x^{\Delta}(t_2)}{c_1} \int_{t_3}^{t} \frac{e_{\frac{-p(s)}{c_1 r(s)}}(s,t_2)}{r(s)} \Delta s$$

By (2.1) we can see that $x(t) \to -\infty$ as $t \to \infty$ which contradicts the assumption $x(t) > 0$ so $x^{\Delta}(t) > 0$ and hence $(r(t)\,\psi(x(t))x^{\Delta}(t))^{\Delta} < 0$ for all large t. Now to prove $(\boldsymbol{ii})$, we define $V(t) = x(t) - tx^{\Delta}(t)$, if there is a $t_3 \in [t_2,\infty)_T$ such that $V(t) > 0$. Suppose this is false, then $V(t) < 0$ on $[t_3,\infty)_T$, thus

$$\left(\frac{x(t)}{t}\right)^{\Delta} = \frac{tx^{\Delta}(t) - x(t)}{t\sigma(t)} = \frac{-V(t)}{t\sigma(t)} > 0.$$

Hence $\frac{x(t)}{t}$ is strictly increasing on $[t_3,\infty)_T$ .Pick $t_4 \in [t_3,\infty)_T$, so that $\tau(t) \geq \tau(t_4)$ for all $t \geq t_4$. Then

$$\frac{x(\tau(t))}{\tau(t)} \geq \frac{x(\tau(t_4))}{\tau(t_4)} := m > 0$$

$$x\left(\tau(t)\right) \geq m\tau(t). \tag{2.4}$$

But from (1.1) we have

$$(r(t)\psi\left(x(t)\right)x^{\Delta}(t))^{\Delta} + p(t)x^{\Delta}(t) = -f\left(t, x^{\tau}(t)\right) < 0.$$

i.e.

$$(r(t)\psi\left(x(t)\right)x^{\Delta}(t))^{\Delta} \leq -f\left(t, x^{\tau}(t)\right) \leq -q(t)x^{\tau}(t) \leq -mq(t)\tau(t)$$

$$r(t)\psi\left(x(t)\right)x^{\Delta}(t) - r(t_4)\psi\left(x(t_4)\right)x^{\Delta}(t_4) \leq -m\int_{t_4}^{t} q(s)\tau(s)\,\Delta s$$

i.e.

$$r(t_4)\psi\left(x(t_4)\right)x^{\Delta}(t_4) \geq m\int_{t_4}^{t} q(s)\tau(s)\,\Delta s$$

This contradicts (2.2). Therefore $V(t) > 0$ for all $t \in [t_3,\infty)_T$, and hence $x(t) > tx^{\Delta}(t)$. Also we see that

$$\left(\frac{x(t)}{t}\right)^{\Delta} = \frac{tx^{\Delta}(t) - x(t)}{t\sigma(t)} = \frac{-V(t)}{t\sigma(t)} < 0.$$

So $\frac{x(t)}{t}$ is strictly decreasing on $[t_3,\infty)_T$.

Theorem 2.2. *Suppose that (2.1) holds. Furthermore, suppose that there exists a positive Δ −differentiable function $\delta(t)$ such that, for all sufficiently large $t_1 \in [t_0,\infty)_T$, one has*

$$\limsup_{t\to\infty} \int_{t_1}^{t} \left\{\frac{\delta(s)q(s)\tau(s)}{\sigma(s)} - \frac{c_2 s r(s) P^2(s)}{4\delta(s)\sigma(s)}\right\} \Delta s = \infty \tag{2.5}$$

where $P(t) := \delta^{\Delta}(t) - \frac{\delta(t)p(t)}{c_2 r(t)}$. Then Eq. (1.1) is oscillatory.

Proof. Suppose to the contrary that $x(t)$ is a nonoscillatory solution of (1.1). We may assume that $x(t) > 0$, $x(\tau(t)) > 0 \ \forall \ t \in [t_1, \infty)_T$. The proof when $x(t)$ is eventually negative is similar. From Lemma 2.1 and Eq. (1.1) it follows that $x(\tau(t)) > 0$, $x^{\Delta}(t) > 0, (r(t)\,\psi(x(t))x^{\Delta}(t))^{\Delta} < 0 \ \forall\, t \geq t_1$. Define

$$\omega(t) = \delta(t)\frac{r(t)\psi\left(x(t)\right)x^{\Delta}(t)}{x(t)}$$

Then

$$\omega^{\Delta}(t) = \delta^{\Delta}\left[\frac{r\psi\, x^{\Delta}}{x}\right]^{\sigma} + \delta\left[\frac{r\psi\, x^{\Delta}}{x}\right]^{\Delta}$$

$$= \frac{\delta^{\Delta}}{\delta^{\sigma}}\omega^{\sigma} + \frac{(r\psi\, x^{\Delta})^{\Delta}}{x^{\sigma}} - \delta\frac{r\psi\,(x^{\Delta})^2}{xx^{\sigma}}$$

From (1.1) and $(\boldsymbol{H_3})$ we have

$$\omega^{\Delta}(t) \leq \frac{\delta^{\Delta}}{\delta^{\sigma}}\omega^{\sigma} - \frac{\delta p}{r\psi}\frac{r\psi\, x^{\Delta}}{x^{\sigma}} - \delta q\frac{x^{\tau}}{x^{\sigma}} - \delta\frac{r\psi\, x^{\Delta}}{x^{\sigma}}\frac{r\psi\, x^{\Delta}}{r\psi x}$$

Since $\sigma > t$, then in view of $(\boldsymbol{H_1})$, and $(r\psi x^{\Delta})^{\Delta} < 0$, we have

$$\omega^{\Delta}(t) \leq \frac{\delta^{\Delta}}{\delta^{\sigma}}\omega^{\sigma} - \frac{\delta p}{c_2 r}\frac{(r\psi\, x^{\Delta})^{\sigma}}{x^{\sigma}} - \delta q\frac{x^{\tau}}{x^{\sigma}} - \frac{\delta}{c_2 r}\frac{(r\psi\, x^{\Delta})^{\sigma}}{x^{\sigma}}\frac{(r\psi\, x^{\Delta})^{\sigma}}{x^{\sigma}}\frac{x^{\sigma}}{x}$$

Since $\frac{x}{t}$ is strictly decreasing and $\tau < \sigma$; $t < \sigma$, then $\frac{x^{\tau}}{x^{\sigma}} \geq \frac{\tau}{\sigma}$, $\frac{x^{\sigma}}{x} \geq \frac{\sigma}{t}$, thus

$$\omega^{\Delta}(t) \leq \frac{\delta^{\Delta}}{\delta^{\sigma}}\omega^{\sigma} - \frac{\delta p}{c_2 r\delta^{\sigma}}\omega^{\sigma} - \frac{\delta q\tau}{\sigma} - \frac{\delta\sigma}{c_2 rt(\delta^{\sigma})^2}(\omega^{\sigma})^2$$

$$= \frac{P}{\delta^{\sigma}}\omega^{\sigma} - \frac{\delta\sigma}{c_2 rt(\delta^{\sigma})^2}(\omega^{\sigma})^2 - \frac{\delta q\tau}{\sigma} \tag{2.6}$$

i.e.

$$\omega^{\Delta}(t) \leq \left[\frac{1}{\delta^{\sigma}}\sqrt{\frac{\delta\sigma}{c_2 tr\delta^{\sigma}}}\omega^{\sigma} - \frac{P}{2}\sqrt{\frac{c_2 tr}{\delta\sigma}}\right]^2 + \frac{c_2 trP^2}{4\delta\sigma} - \frac{\delta q\tau}{\sigma}$$

therefore

$$\omega^{\Delta}(t) \leq \frac{c_2 trP^2}{4\delta\sigma} - \frac{\delta q\tau}{\sigma}$$

Integrating this inequality from t_2 to t, we get

$$\int_{t_2}^{t}\left\{\frac{\delta(s)q(s)\tau(s)}{\sigma(s)} - \frac{c_2 sr(s)P^2(s)}{4\delta(s)\sigma(s)}\right\}\Delta s \leq \omega(t_2) - \omega(t) \leq \omega(t_2)$$

This contradicts with (2.5), hence the proof is completed. By choosing $\delta(t) = 1,\ \ t \geq t_0$ in Theorem 2.2 we have the following oscillation result.

Corollary 2.3. *Assume that the assumptions of Theorem 2.2 hold and for all sufficiently large* t_1,

$$\lim_{t\to\infty} sup \int_{t_1}^{t} \left\{ \frac{q(s)\tau(s)}{\sigma(s)} - \frac{sp^2(s)}{4c_2 r(s)\sigma(s)} \right\} \Delta s \ = \infty$$

Then every solution of Eq. (1.1) is oscillatory on $[t_0, \infty)_T$. By choosing $\delta(t) = t,\ \ t \geq t_0$ in Theorem 2.2 we have the following oscillation result.

Corollary 2.4. *Assume that the assumptions of Theorem 2.2 hold. Then every solution of Eq. (1.1) is oscillatory on* $[t_0, \infty)_T$ *provided that*

$$\lim_{t\to\infty} sup \int_{t_1}^{t} \left\{ \frac{sq(s)\tau(s)}{\sigma(s)} - \frac{c_2 r(s) P^2(s)}{4\sigma(s)} \right\} \Delta s \ = \infty$$

where $P(t) = 1 - \frac{tp(t)}{c_2 r(t)}$.

Now, we define the function space $\mathfrak{R}$ as follows: $H \in \mathfrak{R}$ provided H is defined for $t_0 \leq s \leq t,\ t,\ s \in [t_0, \infty)_T,\ H(t,t) = 0,\ H(t;\ s) \geq 0$ and H has a nonpositive continuous $\Delta -$*partial derivative* $H^{\Delta_s}(t,\ s) \geq 0$ with respect to the second variable and satisfies for some $h \in \mathfrak{R}$. The following theorem extends Theorem 2.2 of [5].

$$H^{\Delta_s}(t,s) \ + \ H(t,s)\frac{P(s)}{\delta^\sigma(s)} = -\frac{h(t,s)}{\delta^\sigma(s)}\sqrt{H(t,s)}$$

Theorem 2.5. *Suppose that the assumptions of Theorem 2.2 hold. If there exists a positive functions* $H,\ h \in \mathfrak{R}$ *such that for all sufficiently large* $t_1 \in [t_0, \infty)_T$ *, one has then every solution of Eq. (1.1) is oscillatory on* $[t_0, \infty)_T$.

$$\lim_{t\to\infty} \sup \frac{1}{H(t,t_0)} \int_{t_0}^{t} \left\{ \frac{\delta(s) q(s)\tau(s)}{\sigma(s)} H(t,s) - \frac{c_2\, s\, r(s) h^2(t,s)}{4\delta(s)\sigma(s)} \right\} \Delta s \ = \infty, \tag{2.7}$$

Proof. Suppose to the contrary that $x(t)$ is a nonoscillatory solution of (1.1). Then as in Theorem 2.2 we have $x(\tau(t)) > 0,\ x^\Delta(t) > 0,\ (r(t)\,\psi(x(t))x^\Delta(t))^\Delta < 0 \ \ \forall\, t \geq t_1 \in [t_0, \infty)_T$. We define $\omega(t)$ as in Theorem 2.2, then from (2.6) we have

$$\frac{\delta q \tau}{\sigma} \leq -\omega^\Delta + \frac{P}{\delta^\sigma}\omega^\sigma - \frac{\delta\sigma}{c_2 r t (\delta^\sigma)^2}(\omega^\sigma)^2 \tag{2.8}$$

Multiplying (2.8) by $H(t,s)$ and integrating from t_1 to t, we get

$$\int_{t_1}^{t} \frac{\delta(s)q(s)\tau(s)}{\sigma(s)} H(t,s)\, \Delta s \leq \int_{t_1}^{t} \omega^\Delta(s) H(t,s)\, \Delta s + \int_{t_1}^{t} \frac{P(s)}{\delta^\sigma(s)} \omega^\sigma(s) H(t,s)\, \Delta s$$

$$- \int_{t_1}^{t} \frac{\delta(s)\sigma(s)}{c_2 r(s) s \left(\delta^\sigma(s)\right)^2} (\omega^\sigma)^2(s) H(t,s)\, \Delta s$$

$$= \omega(t_1)H(t,t_1) + \int_{t_1}^{t} H^{\Delta_s}(t,s)\, \omega^\sigma(s)\, \Delta s \ + \int_{t_1}^{t} \frac{P(s)}{\delta^\sigma(s)} \omega^\sigma(s) H(t,s)\, \Delta s$$

$$-\int_{t_1}^{t} \frac{\delta(s)\sigma(s)}{c_2 r(s) s \left(\delta^{\sigma}(s)\right)^2} (\omega^{\sigma})^2(s) H(t,s)\, \Delta s$$

Thus

$$\int_{t_1}^{t} \frac{\delta(s)q(s)\tau(s)}{\sigma(s)} H(t,s)\, \Delta s \le \omega(t_1)H(t,t_1)$$

$$-\int_{t_1}^{t} \left\{ \frac{1}{\delta^{\sigma}(s)} \sqrt{\frac{\delta(s)\sigma(s)H(t,s)}{c_2 r(s) s}}\, \omega^{\sigma}(s) + \frac{h(t,s)}{2} \sqrt{\frac{c_2 r(s) s}{\delta(s)\sigma(s)}} \right\}^2 \Delta s + \int_{t_1}^{t} \frac{c_2 r(s) s h^2(t,s)}{4\delta(s)\sigma(s)}\, \Delta s$$

$$\le \omega(t_1)H(t,t_1) + \int_{t_1}^{t} \frac{c_2 r(s) s h^2(t,s)}{4\delta(s)\sigma(s)}\, \Delta s$$

i.e.

$$\frac{1}{H(t,t_1)} \int_{t_0}^{t} \left\{ \frac{\delta(s)q(s)\tau(s)}{\sigma(s)} H(t,s) - \frac{c_2\, s\, r(s) h^2(t,s)}{4\delta(s)\sigma(s)} \right\} \Delta s \le\ \omega(t_1)$$

which contradicts (2.7) and hence the proof is completed.

Theorem 2.6. *Suppose that (2.1) holds. Furthermore, suppose that there exists a function* $g(t)$ *such that* $r(t)g(t)$ *is a* $\Delta-$ *differentiable function and there exists a positive real rd-function* $v(t)$ *such that, for all sufficiently large* $t_1 \in [t_0, \infty)_T$ *, one has*

$$\lim_{t\to\infty} sup \int_{t_1}^{t} \{E(s) - \frac{B^2(s)}{4A(s)}\} \Delta s\ = \infty, \tag{2.9}$$

where $A(t) := \frac{v^{\sigma}(t) t}{c_2 r(t)\sigma(t) v^2(t)}$, $B(t) := \frac{v^{\Delta}(t)}{v(t)} - \frac{t v^{\sigma}(t) p(t)}{c_2 r(t)\sigma(t) v(t)} + \frac{2t v^{\sigma}(t) g(t)}{c_2 \sigma(t) v(t)}$, *and* $E(t) := \frac{v^{\sigma}(t) q(t)\tau(t)}{\sigma(t)} + \frac{t r(t) v^{\sigma}(t) g^2(t)}{c_2 \sigma(t)} - \frac{t p(t) v^{\sigma}(t)}{c_2 \sigma(t)} - v^{\sigma}(t)[r(t)g(t)]^{\Delta}$. *Then Eq. (1.1) is oscillatory.*

Proof. Suppose to the contrary that $x(t)$ is a nonoscillatory solution of (1.1). We may assume that $x(t) >\ 0, x(\tau(t)) > 0\quad \forall\, t \in\ [t_1, \infty)_T$. The proof when $x(t)$ is eventually negative is similar. From Lemma 2.1 and Eq. (1.1) it follows that $x(\tau(t)) >\ 0,\ x^{\Delta}(t) > 0,\ (r(t)\, \psi(x(t)) x^{\Delta}(t))^{\Delta} <\ 0\quad \forall\, t \ge t_1 \in\ [t_0, \infty)_T$. Define

$$u(t)\ = v(t)[\frac{r(t)\psi(x(t))x^{\Delta}(t)}{x(t)} + r(t)g(t)]$$

Then

$$u^{\Delta}(t) =\ v^{\Delta}\left[\frac{r\psi\, x^{\Delta}}{x} + rg\right] + v^{\sigma}\left[\frac{r\psi\, x^{\Delta}}{x} + rg\right]^{\Delta}$$

$$= \frac{v^{\Delta}}{v} u + v^{\sigma} \frac{(r\psi\, x^{\Delta})^{\Delta}}{x^{\sigma}} - v^{\sigma} \frac{r\psi\, (x^{\Delta})^2}{x x^{\sigma}} + v^{\sigma}[rg]^{\Delta}$$

From (1.1) and $(\boldsymbol{H_3})$ we have

$$u^{\Delta}(t) \leq \frac{v^{\Delta}}{v}u - \frac{v^{\sigma}p}{r\psi}\frac{r\psi\, x^{\Delta}}{x}\frac{x}{x^{\sigma}} - v^{\sigma}q\frac{x^{\tau}}{x^{\sigma}} - \frac{v^{\sigma}}{r\psi}\frac{x}{x^{\sigma}}(\frac{u}{v} - rg)^2 + v^{\sigma}[rg]^{\Delta}$$

Using Lemma 2.1 and ($\boldsymbol{H_1}$), we get

$$u^{\Delta}(t) \leq \frac{v^{\Delta}}{v}u - \frac{v^{\sigma}pt}{c_2r\sigma}\left(\frac{u}{v} - rg\right) - \frac{v^{\sigma}q\tau}{\sigma} - \frac{v^{\sigma}tu^2}{c_2r\sigma v^2} + \frac{2v^{\sigma}tu}{c_2\sigma v} - \frac{v^{\sigma}trg^2}{c_2\sigma} + v^{\sigma}[rg]^{\Delta}$$

i.e.

$$u^{\Delta}(t) \leq \frac{v^{\Delta}}{v}u - \frac{v^{\sigma}pt}{c_2r\sigma v}u + \frac{v^{\sigma}ptg}{c_2\sigma} - \frac{v^{\sigma}q\tau}{\sigma} - \frac{v^{\sigma}t}{c_2r\sigma v^2}u^2 + \frac{2v^{\sigma}tu}{c_2\sigma v} - \frac{v^{\sigma}trg^2}{c_2\sigma} + v^{\sigma}[rg]^{\Delta}$$

$$= -Au^2 + Bu - E = -[\sqrt{A}\,u - \frac{B}{2\sqrt{A}}]^2 + \frac{B^2}{4A} - E$$

i.e.

$$u^{\Delta}(t) \leq \frac{B^2(t)}{4A(t)} - E(t)$$

By integrating the above inequality, we obtain

$$\int_{t_2}^{t}\{E(s) - \frac{B^2(s)}{4A(s)}\}\Delta s \leq u(t_2) - u(t) \leq u(t_2)$$

This contradicts (2.9), and the proof is completed. We can get the following result by choosing $v(t) = 1$ in Theorem 2.6.

Corollary 2.7. *Assume that the assumptions of Theorem 2.5 hold and for some* t_1 *sufficiently large . We have*

$$\limsup_{t\to\infty}\int_{t_1}^{t}\{E(s) - \frac{B^2(s)}{4A(s)}\}\Delta s = \infty,$$

where $A(t) := \frac{t}{c_2r(t)\sigma(t)}$, $B(t) := -\frac{tp(t)}{c_2r(t)\sigma(t)} + \frac{2tg(t)}{c_2\sigma(t)}$, *and* $E(t) := \frac{q(t)\tau(t)}{\sigma(t)} + \frac{tr(t)g^2(t)}{c_2\sigma(t)} - \frac{tp(t)g(t)}{c_2\sigma(t)} - [r(t)g(t)]^{\Delta}$.

Then Eq. (1.1) is oscillatory.

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